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FEEDBACK LOOP SHAPING FOR ACTIVE NOISE CONTROL WITH CONSTRAINTS OF THE WORST-CASE SECONDARY PATHS

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Abstract: *Active Noise Control is a vast field in scientific research. The optimization of the open or closed loop of a feedback system is well explored and several optimization approaches are found in literature. It is common to measure the secondary path, i.e. the transfer functions of the microphone and the loudspeaker in an ANC-headphone system, via sweeps, which can be used as a reference model for the optimization of a feedback system. Although the system is optimized, the misuse of the headphone, i.e. a leak between the ear and the headphone cushion or pressing the ear cup against the head, can lead to instability. For an effective reference model, several measurements have to be made. Especially the worst-case scenarios have to be taken into account. It is possible to determine different secondary paths for different measurement positions on the listener's head. The combination of worst-case secondary paths, which are transfer functions of leaky and pressed down measurements, can be used as a set of worst-case constraints for closed or open loop optimization. Within this contribution a method of defining worst-case constraints is presented. This method optimizes the open or closed loop of a regular secondary path to maximize the ANC-performance and it never violates the stability constraints.*

Keywords: ANC, optimal filter design, feedback loop shaping, plant uncertainty, headphones, secondary path models, worst-case feedback constraints

1. INTRODUCTION

The first patent for a active noise control (ANC) system is dated back to the year 1936 [1]. Ever since, the control of noise in different environments has been explored. On the one hand improvement in better damping of the housing of a radiating noise source and on the other hand hearing protection can be used as a salvation for noise problems. In some cases only the latter one is possible. That is why active noise control is used to reduce ambient noise in modern headphones. The two main approaches for ANC headphones are feedback and feed forward noise control. In this paper we will emphasize on the feedback approach, the optimization of the control filter and the plant uncertainty for secondary paths of headphones.
In classic control theory the H_∞ and H_2 approaches cf. [2, 3] are used to optimize a feedback loop. Most of the classic controller synthesis methods need a translation of a physical system into a state-space plant model. Furthermore, proper weighting functions and uncertainty models need to be designed to achieve good results. Because this is often impractically newer investigations on feedback ANC-headphones have been made [4, 5]. They use frequency responses calculated from raw measurement data (i.e impulse response measurements) of the headphones instead. In these approaches, a nominal frequency response is used to calculate the optimal feedback filter formulated as a linear [4] and as a nonlinear [5] optimization problem with constraints to ensure closed loop stability.
We seek for the definition of worst-case wearing scenarios (e.g open, leaky, pressed etc.) and their importance in stability terms. Different wearing scenarios require different constraints, which can be attested in the frequency domain. This plant uncertainties can be easily measured and taken into account in the process of synthesizing an optimal feedback filter.

2. FEEDBACK CONTROL AND OPTIMIZATION

In the case of a feedback ANC, the superposition of the penetrating ambient noise and the played back anti-noise are picked up by a microphone inside the ear-cups. The residual noise signal is sent to the feedback loop and shaped by a control filter. The approach is also referred to as closed loop shaping in control system theory, whereas the control filter

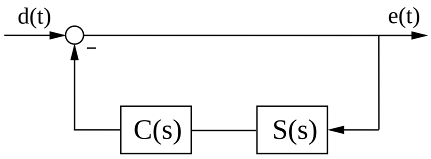

Fig. 1. Block diagram of the closed feedback loop, where C(s) describes the filter to shape S(s) (i.e. the secondary path) to form an stable closed loop.

ensures stability. Due to the stability constraints , the filter limits the bandwith. Active noise control thus reduces low frequency noise, while high frequency noise is attenuated due to the passive damping of the headphone ear-cups.
Fig. 1 shows the block diagram of a feedback loop, where S(s) is referred to as the secondary path (i.e the transfer function from the loudspeaker to the microphone), C(s) is the transfer function of the filter, d(t) are the noise disturbances and e(t) denotes the residual error signal. An optimal noise reduction system would achieve an error equal to zero, which is not possible in reality. Due to the typical phase delay of the mechanical system, high frequencies can't be cancelled as good as low frequencies. Furthermore, a noise reduction in one frequency region introduces unavoidably an amplification in another region, which is known as the waterbed-effect [6].
The achieved noise rejection can be shown via the magnitude response of the closed loop transfer function (error over disturbances relationship), which is known as the sensitivity function and denoted as follows

$$T(s) = \frac{E(s)}{D(s)} = \frac{1}{1 + C(s)S(s)} = \frac{1}{1 + L(s)}, \quad (1)$$

where $L(s)$ describes the open loop. If we now want to minimze the sensitivity function in a specific frequency region we can denote an objective function J that depends on a nominal secondary path S_n

$$\min \; J(S_n(s)),$$
$$\text{with subject to the constraining function: } f(\mathcal{S}) < 0, \quad (2)$$

whereas S_n represents the secondary path of the intended wearing scenario and $\mathcal{S}$ holds all possible secondary path variations (i.e. plant uncertainty) to constrain the optimization process.

3. CONSTRAINTS

In general, we can define two main restrictions to guarantee stability. One of the most critical constraints in feedback filter design is the phase margin. The phase response of the open loop has to be within the region of π and $-\pi$ for the whole frequency band where the magnitude is above unity. Furthermore, the phase margin in real environments is chosen even more conservative as mentioned to prevent excessive noise amplification.
The second most critical constraint is the gain margin which is closely related to the phase margin. The gain in the frequency band of the open loop where noise reduction is performed should be large. If the phase of the open loop reaches $-\pi$ the magnitude should be below unity to prevent noise amplification [7]. In most commercial products, the noise amplification margin for $T(s)$ is around 4dB as in Eq. (3)

$$20 \log \left| \frac{1}{1 + L(s)} \right| \leqslant 4\text{dB}. \quad (3)$$

This constraint has two degrees of freedom, namely the magnitude and phase of the open loop. In manually shaped loops typically a phase margin of at least 40° is demanded. This means that the phase of the open loop has to stay above -140°($\widehat{=} -\frac{5\pi}{6}$) in the frequency band of interest. The maximum open loop gain α can consequently be calculated over

$$20 \log \left| \frac{1}{1 + \alpha e^{-j\frac{5\pi}{6}}} \right| \leqslant 4\text{dB}. \quad (4)$$

These two necessary restrictions prevent that the frequency response locus of $L(s)$ encircles the critical point -1. In [4], the magnitude and phase margins are expressed as linear constraints for a convex optimization problem in the cepstral domain. Whereby [5] uses nonlinear constraints for a nonconvex optimization problem. These can be stated to shape the open loop not only via the phase and gain margin, but also with forbidden regions in the nyquist plot. However, neither of the references explicitly investigate the variations in the secondary path, which has a noticeable impact on the control filter as will be shown in the following.

4. WORST CASE SCENARIOS

The variations in the secondary path depend on the fitting of the headphones. Fig. 2 shows measurements of a normal wearing position (onhead), a scenario where the headphone is slightly off the normal position (loose), the case before the headphone is put on (open) and the case when the headphone is pressed against the ears. It is possible to set up worst-case constraints (SYN), if we take the envelope of the highest magnitude and the phase values closest to -$n\pi$ for n $\in \mathbb{N}$.
As stated in Eq. (2) the constraining function accounts for several secondary paths, and the outcome of the filter optimization depends on which wearing scenarios are included in $\mathcal{S}$. If one wants to enhance the optimization process to variation in the secondary path it is advisable to include all misuse scenarios in the constraining function to provide stability under all circumstances. However, not every misuse scenario has the same impact on the phase and gain margins. It is sufficient to consider the worst case frequency responses which have the phase values closest to -$n\pi$ and those with the largest amplitude responses (worst-cases).
As mentioned above, one can synthesize a single frequency response out of the worst case magnitude and phase values per frequency bin in order to provide computational efficiency. Note: this synthesized frequency response does not represent a real physical secondary path.
From Fig. 2 it can already be seen that the secondary path of

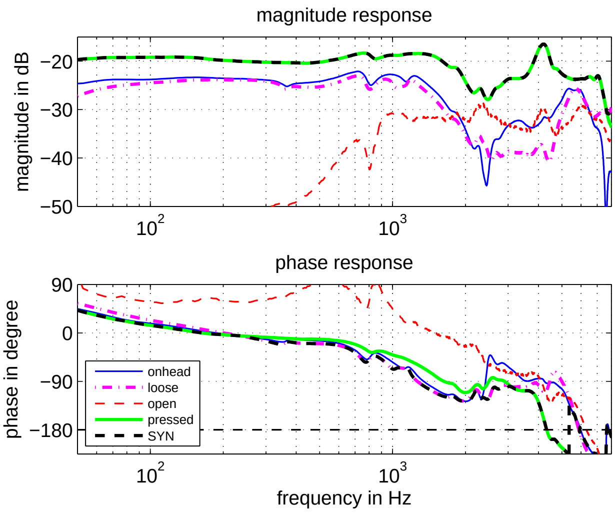

Fig. 2. Measurement scenarios: The case when the headphone is pressed against the ear yields a frequency response with the largest magnitude and a fast falling phase. The opposite result is seen when the headphone is not put on (open). It is not possible to use only one measurement to define an overall worst-case, although we can construct a synthesized frequency response out of the worst-case pieces of all measurements (SYN).

the pressed headphones has the largest amplitude response and represents the worst-case for almost the whole bandwidth. Due to the increased magnitude of the pressed secondary path, the amplification of the control filter is necessarily smaller than it would be for the nominal secondary path. This can be observed in Fig. 3 which depicts the outcome of the optimization for different secondary path measures in the constraining function. It can also be seen that the synthesized

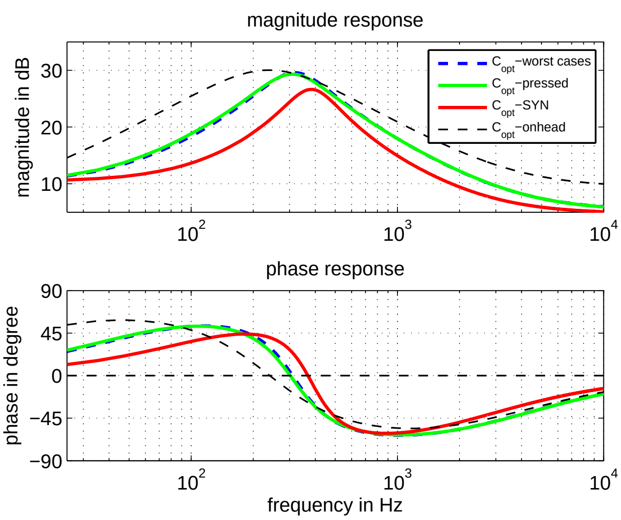

Fig. 3. Outcome of the optimization for 2nd order optimal filters with different secondary paths in the constraining function. Synthesized constraints result in the most conservative optimal filter, whereas an optimization on the nominal secondary path leads to the opposite.

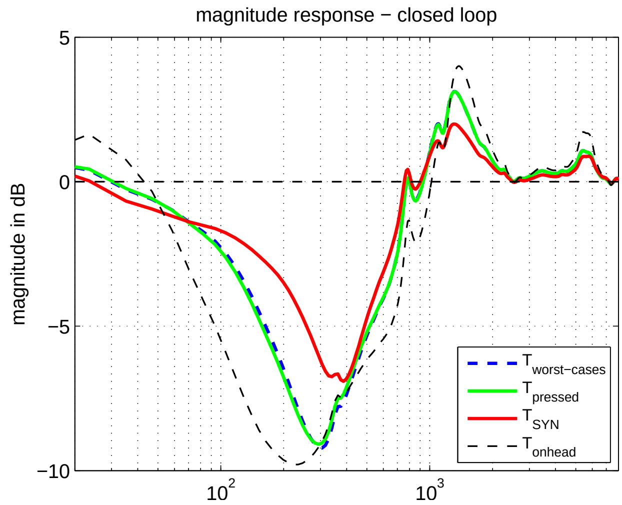

Fig. 4. Performance analysis of 2nd order optimal filters in Fig. 3 applied to the nominal secondary path (i.e standard fitting of the headphone). The design constraint for the maximum amplification restricts the magnitude to be smaller or at least equal to 4dB.

secondary path leads to a more conservative control filter in comparison to the more processing intense optimization task which uses all physical worst-case secondary paths.

5. PERFORMANCE ANALYSIS

The performance of ANC can easily be assessed via the sensitivity function $T(s)$. For the first performance analysis we use the nominal secondary path together with the optimal filters $C_i(s)$ presented in Fig. 3 yielding

$$T_i(s) = \frac{1}{1 + S_n(s)C_i(s)}. \quad (5)$$

Fig. 4 shows that the filter designed for the nominal plant yields more noise reduction between 40Hz and 1kHz, but also gets closest to our maximum amplification design constraint for the sensitivity function, which is $20 \log \max |T(s)| \leqslant 4$dB. As already mentioned in Sec. 3, we observe an amplification of higher frequencies in the trade-off to a good noise rejection in the lower frequency region.
In the second performance analysis we use the pressed secondary path S_{press} to investigate the sensitivity function

$$T_i(s) = \frac{1}{1 + S_{\text{press}}(s)C_i(s)}. \quad (6)$$

In this case, the optimal filter designed for the nominal secondary path C_{onhead} violates the magnitude design constraint of $T(s)$ and causes a noise amplification of more than 7dB as can be seen in Fig. 5. All other filters meet the constraints, whereas the most conservative filter C_{SYN} achieves the lowest performance. Since S_{press} represents the worst case secondary path for almost the whole frequency range,

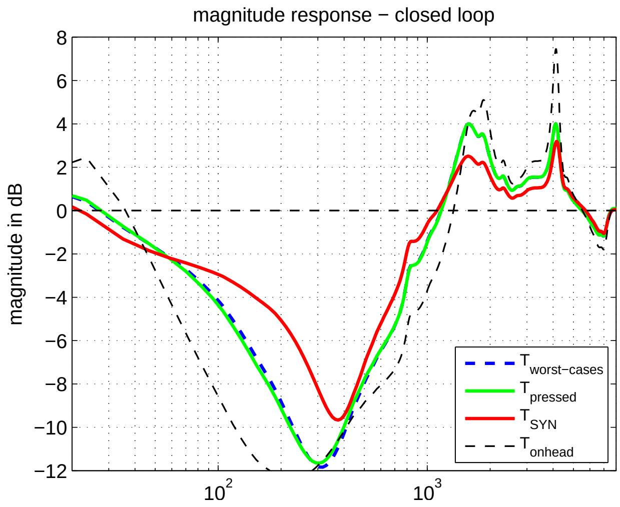

Fig. 5. Performance analysis of 2nd order optimal filters applied to the pressed secondary path. The filter C_{onhead} designed only for the standard wearing situation violates the design constraint (i.e. $20 \log \max |T(s)| \leqslant 4$dB).

the results of T_{press} and T_{wc} (includes all worst-cases) are very similar.

6. CONCLUSION

In feedback ANC headphones, the optimal control filter depends on the secondary path. The secondary path of a usual wearing situation is regarded as the nominal secondary path. However the secondary path changes considerably if the headphones are lifted or pressed against the ears [8]. It is therefore not sufficient to consider only the nominal secondary path in the constraints of the feedback filter optimization. To fulfill the constraints for all usage scenarios, the secondary paths which represent the worst-cases have to be considered.

Further we can denote that it is necessary to include all worst-case constraints if we want to guarantee stability. From the magnitude and phase constraint for the control filter, it follows that the worst-case secondary paths are the ones which either show the largest magnitude or whose phase response is closest to $-n\pi$ in some bandwidth.

It has been shown that the secondary path of the pressed headphones represents the worst-case in a broad bandwidth, but in general it has to be assumed that more than one secondary path measure has to be considered in the constraints of the filter optimization. This is especially true for the middle and high frequency band, where the phase of the shaped open loop crosses π several times.

In order to save computational time, one could synthesize a single frequency response that combines piecewise worst-case magnitude with worst-case phase responses (from all measured secondary paths). Consequently, this abstract frequency response gives more conservative constraints for the optimization task and yields a lower noise canceling performance. Since the optimization task hast to be done only once and off-line, it is preferably to spend more optimization time and to consider all worst-case secondary paths because the resulting performance can be increased by 2dB in almost the complete ANC bandwidth.

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